IASS-Symposium on Practical Aspects in the Computation os Shell and Spatial Structures[1]

A. SAMARTIN QUIROGA, M. VAN LAHEM AND G. DE ROECK

1. Numerical Methods and Numerical Modeling

1.1 **W.C.Schnobrich** reviews the different methods of analysis of shells: famous names appear in the historical aspect, including finite difference methods and numerical integration. The finite element models are evaluated and the state of the art precised.

A list of 44 references, some explicitely making mention of reinforcement, material nonlinearity, time dependent effects, dynamics, etc., completes this opening paper.

1.2 **M.Bernadou** exposes theory of finite element methods for linear thin shell problems referring to KOITER 's work. Concepts such as image of a set by mapping are readily used.

The 28 references concern amongst other topics: the nonlinear theory of thin elastic shells, also the convergence of finite element methods, computer procedures, a stiffness matrix for shallow rectangular shell and the discrete Kirchhoff triangle, to mention only a few.

1.3 **Sardar Amin Saleh** presents a computer program for an automatic design of shell structures and its numerical processing. Computer aided design and artificial intelligence are thus introduced into shell design.

1.4 **Fan Yashen** links the elastic thin shallow shell's governing equations to the Helmholz equation derived from the theory of plane wave propagation. For shells with circular bottom and also shells with rectangular bottom, the deflection and the stress function (Airy, Pucher) are obtained as series with constants to be determined by the boundary conditions of the two functions.

References concern, besides shells: mathematical physics, partial differential equations and higher transcendental functions.

1.5 **M.Kurata** treats bond slip equations of reinforced concrete shells. The nonlinear basic equations of the reinforced concrete shells include the relation between the bond shear stress and the relative slip of the bar after deformation. A finite element formulation results from the application of the incremental virtual displacements. The geometrical nonlinearities of bond slip have an influence on the elastic-plastic behavior of reinforced concrete shells.

Two of the references concern bond slip, seven treat nonlinear behavior and one also cracking j1.6 The ATTILA code was developed for the French Navy and comprises a finite element modelling taking into account the assembling of three-dimensional and shell elements, the piezoelectric effect, the fluid-structure interaction and the radiation damping.

B.Hamonic, J.C.Debus, J.N.Decarpigny, D.Boucher and **B. Tocquet** present a detailed analysis of the acoustic behavior of a new flexural shell sonar transducer.

Experimental and numerical results are compared.
Of the seventeen references, two concern acoustics, three the finite element modelling of transducers, one the ATTILA code.

1.7 **P.L.Gould, M.Hourani, J.S.Lin, T.G.Harman** and **K.J.Han** apply four finite element computer programs to analyse thick cylindrical shells (R/t=12.5) under concentrated ice pressure loading which might produce a punching shear failure.

"The objective of the study was to determine the most appropriate support conditions for a reinforced concrete shell panel test specimen which will adequately reproduce internal forces and moments in a prototype cylindrical shell."

The results are satisfactory and are thouroughly discussed. "It is shown that various thick shell and solid elements can be satisfactorily employed in the vicinity of the loading, together with less involved thin shell elements over the remaining domain of the shell."

Currently is in development a three-dimensional smeared non-orthogonal cracking model, including bond-slip and tension stiffening.

Besides the principal books and papers by two of the authors, the references concern ANSYS and PDA/PATRAN.

1.8 **San-Cheng Chang** and **Jhin-Hwei Lee** present a curved stiffener finite element compatible with regard to the nodal degrees of freedom with a doubly-curved triangular thin-shell finite element, which was jointly developed by
S. C. Wu and San-Cheng Chang.

The shell element has three corner nodes and three mid-side nodes. In each corner node the nine d.o.f. are the displacements in the principal curvature directions and the normal, and their first derivative in each of these three orthogonal directions. In each mid-side node the three d.o.f. are the first derivatives in the direction of the surface tangent vector, located at mid-side node normal to
the element edge, of the following three displacements: the first one in the direction of the unit tangent vector at the edge, the second in the direction in which are taken the derivatives and the third in the normal direction.

The stiffener element has three nodes and 21 d.o.f. It can be located arbitrarily on a free-form surface defined by a set of user chosen curvilinear coordinate system.

Both elements satisfy the rigid-body mode requirement.

Three examples are reported: the free vibration of a stiffened shell, the snap-through of a stiffened spherical cap and the linear static and free vibration behaviors of a stiffened parabolic shell.

Reference topics: laminated anisotropic curved beam; geodesic beam; integrated finite element nonlinear shell analysis system with interactive computer graphics; free vibration of ring-stiffened cylindrical shells.

---

recommendations for air-supported structures

Prepared by IASS Working Group N.º 7. **"Pneumatic Structures"**
Chairman: Prof. Y. Tsuboi

CONTENTS:

This 213 pages IASS publication is available from:

IASS Secretariat
Alfonso XII, n.º 3
28014 Madrid (Spain)

Price (Surface mail included): 40 Swiss Francs.

Extra cost for air mail:

Europe	7 Swiss Francs
America, Africa, Middle East, India	15 Swiss Francs
Asia and Oceania	20 Swiss Francs

Way of payment: **Bank transference to:**
Guipuzcoano Bank,
IASS account n.º 11.758,
Paseo de la Castellana, 77.
28046 Madrid (Spain).

If a transfer is not used as way of payment, the amount of the cheque should be increased in 10 S. Fr. to cover bank operations.

NOTE: Your order must be accompanied by a copy of the corresponding transference.

2. Spatial Structures

2.1 M.**Papadrakis** reviews the static analysis methods for spatial structures. Formex algebra is a breakthrough in their algebraic representation and processing. Numerical methods of nonlinear structural mechanics can provide the necessary effective tools for the analysis of simultaneous plastic collapse and buckling behavior in a large displacement environment.

The discrete field analysis considers mathematical models consisting of difference equations. The equivalent continuum analysis on the contrary, approximates either a system of difference equations by differential equations (by means of Taylor's expansions) or the structure itself by a continuum. The finite element method is the only one having almost unlimited capacity for treating material and geometric nonlinearities. Experiments can verify analysis and lead to discovery of phenomena which leave behind existing approaches.

Collapse analysis is either plastic mechanism approach, elastic stability approach or numerical approach. The plastic mechanism approach accomodates stability effects very approximatively. The elastic stability approach should include several factors, for instance the deformed geometry immediately prior to buckling. To include thereby plastic yielding is very approximately possible by reducing the modulus of elasticity. The numerical approach is the only one which can predict the inelastic post-buckling response because all nonlinearities can fully be taken into consideration.

Incorporating any member in the structural response requires analytical simplified expressions of its axial and/or flexural stiffness at each displacement level based on theoretical and experimental results.

The general 3-D nonlinear beam formulation is not a simple extension of a 2-D formulation because large rotations are not vector quantities (vector summation being commutative). This and other items make the development of general nonlinear beam elements most challenging. An isoparametric beam model needs almost five times more computational time as an Hermitian model but, unlike this latter, is compatible with isoparametric plate and shell elements.It also can better model curved line elements and is recommended with total Lagrangian formulation when considering material nonlinearities. The natural 3-D beam element is based on the natural mode techniques, backboned by the subelement concept. The noncommutative nature of rotations is treated with commutative rotations called semitangential torques. All plastic effects are concentrated at the ends. In comparing the computation time it was found that the natural element can need only a tenth of the CPU time required for the computation with a 50 layered element.

To model the effects of material yield a "section" or a "fibre" type of models may be considered.

Other 3-D beam models could be found in the numerous references.

Several algorithms for the solution of the equations are available.

The linear solution techniques serve even in nonlinear and dynamic analyses at intermediate stages. The iterative solution technique outruns the direct solution techniques because of the widespread use of the minicomputers and the special features of the new generation of parallel computers. The iterative methods could be performed on the "element" level.

Nonlinear solution techniques are iterative, incremental or both. The principal formulas of the following methods are shown and illustrated.

A) Elimination methods

-Successive substitution methods (Newton iteration)

-NR (Newton-Raphson) method

-mNR (modified Newton-Raphson method)

-Matrix update methods or quasi-Newton methods as the BFGS (Broydon-Fletcher-Goldfarb-Shanno) method. These compromises between the NR and the mNR are secans methods.

B) Vector iteration methods, based on the three-term recursion formula, evaluating $U^{(i+1)}$ as function of $f^{(i)}$, $U^{(i)}$ and $U^{(i-1)}$ using a couple of iteration coefficients.

Such methods require only four vectors to be stored.

Combining direct and iterative numerical methods are: a secant Newton method based on the variable metric advancement, the conjugate-Newton method and modified conjugate-Newton, and the preconditioned CG methods.

Effective for highly nonlinear problems are the coupling between Newton and quasi-Newton methods and the use of line searches, because the solution vector can thus be scaled down or damped.

At this point of the considered paper, the title "Mechanical Qualification of Large Flexible Spacecraft Structures" of the AGARD Conference Proceedings No. 397 comes in mind. Paolo **Santini**, chairman, writes in the preface:"... The discussion pointed out some of the pitfalls associated with the greatly increased use of advanced methods of experimental and theoretical dynamic analysis which are now becoming available..."

Finally, M. Papadrakis considers the load-deflection path with snap-through and snap-back buckling. Although the complete curve is quite unneccessary for most pratical problems, M. Papadrakis notes that "...collapse loads are often associated with a failure to achieve convergence with the iterative solution procedure." It is important to know the load shedding after a limit point. This applies to a structure part in order to assess the performance of the complete structure. The circumvention of the limit points by the displacement incrementation displacement control methods succeeds in case of snap-through but fails in case of snap-back (to a lower load level). To overcome this, the load steps are limited by constrained equations, different cases of which are considered.

The application of the considered solution procedures of nonlinear equations to the limit state analysis and post-buckling behavior of pin-jointed spatial trusses can be replaced by simplified methods, some of which are shortly explained and several put in the references.

A list of 83 references closes the considered basic paper 2.1 of Session 2.

2.2 **H.Tabar-heydar** presents a method which, in the case of large scale space frames with pin-jointed truss elements, is preferable above methods based on the direct solution of the equations of equilibrium, except in rare pathological cases.

The total potential energy as a function of nodal displacements is minimized by the Conjugate Gradients algorithm. The progress of the algorithm can be monitored, by printing of the scalar quantities (total potential energy, norm of the out of balance force) after each iteration. Examples of well known big structures are given.

The primary storage requirements are limited to a few vectors making the algorithm particularly attractive for implementation on micro or small minicomputers. Substantial economy of analysis and the facility to monitor the progress of the analysis and obtain valuable information long before the analysis is completed are paramount advantages.

The six references comprise topics as conjugate gradients, minimisation, convergence and finite elements.

2.3 **Z.S.Makowski, D.Maranzano** and **G.A.R.Parke** put the attention on the efficient structural behavior of single-layer diagonal grids and the co-operative action of all the grid members, specially in the case of full continuity of the interconnected beams (electric arc welding). Record clear spans (217m) are reported. The accomodation of heavy concentrated loads is also cited.

Ten different cases of boundary conditions of a square grid are put successively in the same system of linear simultaneous equations. The formex algebra (Nooshin) is used in generating the grid topology and in the post-processing.

In the comparison of results, two conclusions are emphasized. First, the great improvement in the stress distribution and the great reduction of displacements obtained through the movement of corner supports to the middle points of the edges. Second, the very uniform stress distribution in the case of columns supporting the grid at four inner points. This case suits the petrol stations very well, the central part being occupied by an office where the four outside portions allow free access to cars.

The references cover modern grid structures, behavior of square diagonal grids, the BOAC hangars at London Airport and the Formex configuration processing.

2.4 **J.P. Coyette** and **P.Guisset** developed a software for geometrically highly nonlinear structures such as the extensible cable networks. Their equilibrium shape and dynamic behavior can be analysed.

The total potential energy is expressed and developed in terms of finite elements including the potential of forces in the three dimensions. The considered total potential energy is expressed as a function of the nodal coordinates. The possibility of introducing boundary conditions such as rolling-ends or contact without friction problems is emphasized.

MINOS 5.0 is the used optimization code: it uses a reduced-gradient algorithm in conjunction with a quasi-Newton algorithm. These algorithms and their implementations are described in one of the referenced publications.

When the static solution is obtained, a dynamic analysis can be executed. The equation of motion is

$$Mp + Kp = F$$

where M is the mass matrix, K the stiffness matrix, F the nodal load vector and p the nodal coordinates vector. The gradient of total potential energy at time t is

$$G = Kp - F$$

For given initial conditions, this equation is directly integrated at time t with a difference method.

Four Fortran computer programs have been written to form a complete package for static and dynamic cable analysis. Results of the dynamic analysis are graphically represented by an animation program.

Amongst the applications is the dynamic analysis of the cable lifted at one end.

Eight references cover the following topics: general purpose graphics system; large deformation static and dynamic finite element analysis of extensible cables; implicit-explicit finite elements in transient analysis (implementation and examples); pretensioned cable roofs; large scale linearly constrained optimization; MINOS 5.0 Users's Guide; cable structures and variational methods in elasticity and plasticity.

2.5 **H.Nooshin** and **P.G.King** introduce FORMEX ANALYSIS, the method of algebraic data generation. The concepts are introduced by means of examples applying to a particular double-layer grid structure. The plain statement appeals to further introduction by means of the three references. They concern configuration processing, introduction to Formion, (i.e. an interactive programming language for data generation using the concepts of Formex Algebra) and Formex and Plenix structural analysis.

3. Dynamic analysis of shell and spatial structures

3.1 A.Samartin and J.Martinez present a practical approach for Levy type solutions of natural frequencies of translational shells and outline an extension to the study of forced vibrations. The curved plate shallow linear elastic thin shell theory is assumed.

The accompanying orthogonal curvilinear coordinates comprise the lineas $(\alpha_1 \alpha_2)$ which are curvature lines.

The introduction of the stress function Φ transforms the equilibrium equations for translation, following α_1 and α_2, into identities and replaces the normal forces and the shearing force by second derivatives of Φ in the further elaboration of systems of equations. The remaining stress resultants are the two bending moments, the twisting moment, the two transverse forces (Q_1, Q_2) and also

$$R_1 = Q_1 + M_{12,2} \text{ and } R_2 = Q_2 + M_{2,1}$$

These seven stress resultants are all expressed in terms of the derivatives of the normal displacement w.

Other classical equations are transformed and put with the foregoing into the equations of equilibrium and the compatibility equations such that finally two governing differential equations of the fourth order in Φ and in w are elaborated.

The Levy process for the static case is first described and presented in a matrix formulation.

For a rectangular planform, the boundary conditions are $N_{11}=0$, $u_2=w=0$ and $M_{11}=0$. The solutions for w and Φ are set to be infinite series. Each term of these is the sum of a particular solution and a complementary solution. The particular one is obtained from the Fourier expansion coefficients of the external load (X_1, X_2 and Z). The complementary solution is the general solution of the homogeneous equations. The Ambartsumyan function W is defined by $w=\nabla^4 W$ and $\Phi=-Eh\nabla_k^2 W$ *where* ∇^2 *is the Laplace* operator $\partial^2/\partial\alpha_1^2 + \partial^2/\partial\alpha_2^2$ *and*

$$\nabla_k^2=K_2\,\partial^2/\partial\alpha_1^2+K_1\,\partial^2/\partial\alpha_2^2.$$

The introduction of these in the equations reduces the one into an identity and the other into a partial differential equation of the eight order. In the case $K_1=K_2$, the Ambartsumyan function is replaced by the Mishonov function W defined as $^2w= {}^2W$ and $^2\Phi=-EhKW$.

Such auxiliary functions are expanded in Fourier series. Thus their introduction into the considered partial differential equation lets replace this by an infinite number of ordinary equations with constant coefficients. The solution of any such equations comprises eight arbitrary constants. These are determined by four kinematic boundary conditions (u_1, u_2, w, $w_{,2}$) and four static boundary conditions (N_{12}, N_{22}, R_1, M_{22}).

The Levy method is extended to the study of small, normal, free vibrations with time-dependent, homogeneous boundary conditions. The external loads are $X_1=X_2=0$ and $Z=-\rho h\partial^2 w/\partial t^2$ where ρh is the mass per unit area.

The differential equation for the Ambartsumyan function W becomes thus

$$\nabla^8\overline{W} + \frac{Eh}{D}\nabla_k^4\overline{W} + \frac{\rho h}{D}\nabla^4\frac{\partial^2\overline{W}}{\partial t^2} = 0$$

in which is put $\overline{W}=W(\alpha_1,\alpha_2)e^{i\omega t}$ $(i=\sqrt{-1})$

The solution of the thus obtained differential equation for W is of the form

$$W(\alpha_1\alpha_2)=\sum_{m=1}^{\infty} W^{(m)}(\alpha_1\alpha_2)$$

where $W^{(m)}(\alpha_1\alpha_2)=F^{(m)}(\alpha_2)\cdot\sin\lambda_m\alpha_1$, $\lambda_m=m\pi/l_1$

For $F^{(m)}(\alpha_2)$ is thus obtained an ordinary differential equation with constant coefficients the characteristic equation of which is solved algebraïcally. The general solution $F^{(m)}(\alpha_2)$ is presented as a finite serie which terms comprise exponential and trigonometrical functions. To a matrix form of solution, boundary conditions in matrix form are associated in order to obtain a system of eight linear simultaneous equations , whose non-trivial solutions are the roots of the equation expressing that the determinant of the matrix of the coefficients is zero. This equation is transcendental and its roots give all the natural frequencies corresponding to m or number of half waves along the α_1 direction. Reference to a corresponding computer program in FORTRAN IV language is given.

Because the described method is difficult, an alternative method is also presented. In this alternative method, the shell is divided into N elements by the lines $\alpha_2=0$, l_2/N, $l_2/2N$, ... $l_2/(N-1)N$, l_2/NN. For each of those shell elements and each Fourier term m, it is possible to obtain a relation between the kinematic boundary values $\lfloor \underline{p}_1\ \ \underline{p}_2 \rfloor$ and the static boundary values $\lfloor \underline{d}_1\ \ \underline{d}_2 \rfloor$:

$$\begin{bmatrix} \underline{p}_1 & & \underline{d}_1 \\ & = \underline{k} & \\ \underline{p}_2 & & \underline{d}_2 \end{bmatrix}$$

in which $\underline{k}$ is the stiffness matrix that can be computed for the case wherein no loads are applied. A mass matrix is evaluated as an integral in which appears a shape function. By assembling the shell elements the discretized dynamic equilibrium equations for the total shell can be derived, i.e. $\underline{M}\ddot{\underline{D}}+\underline{K}\underline{D}=\underline{0}$ yielding the undamped free vibrations.

Damping effects can be introduced by assuming an orthogonal mass matrix.

The extension to the study of forced vibrations requires the introduction of the applied loading, i.e. for each shell the nodal equivalent force vector.

A first example concerns a cylindrical shell roof and a second a simply supported concrete box bridge. The results are compared with results obtained by means of other methods.

The main conclusion is that the Levy solution of analysis of translational shells can be extended to dynamic situations, but that the actually presented modification is very advantageous.

References concern dynamic analysis of translational shells and dynamics of structures.

3.2 **T.Tamani** and **Y.Hangai** investigate the dynamic behaviors of reticulated single-layer shallow domes by means of response observations of a reticulated single layer model under the natural earthquakes as well as the free vibration tests and the shaking table tests for another model with the same configuration and dimensions.

A model of reticulated single-layer shallow dome triangularly meshed by means of pin-connected, brass truss members, is submitted to an axisymmetrical load by cutting of a string which supported the weight of 80N at the crown. Its accelerations and the surrounding joints move the contrary direction, proving the axisymmetric behavior, which is composed by the corresponding free vibration motions which were computed in advance. In an analogous way an asymmetrical test was done and compared to the computed free vibrations.

The earthquake responses of the same model are investigated by using the up-and-down excitations of various earthquake records, as for instance the natural earthquake wave recorded at the Chiba Experiment Station of the Institute of Industrial Science, University of Tokyo, on October 4, 1985. When exciting, the time scale of input data is condensed in one-fifth of real time. The Fourier spectra of response acceleration and the spectrum ratios are obtained by making the ratios to the values proceeding from the accelerometer on the base of the model. These spectra have roughly the characteristics of the spectra deduced from the natural frequencies of the model.

The free vibration tests and the shaking table tests show that the considered small model has the asymmetrical and the symmetrical mode shapes with about 75Hz and over 90Hz, respectively. These values correspond to the results from the free vibration analysis.

In order to get data under the real earthquakes, a model is constructed on the second floor of a concrete tower. The outline of the plan of the earthquake observation is introduced. The dynamic soil-structure interaction of the reinforced concrete tower will be observed.

Reference topics: state-of-the-art report on space frames, seismic response of a spherical shell and the mentioned observation of soil-structure interaction.

3.3 **J.Monbaliu** considers wind-structure interaction by joining atmospheric turbulence to dynamic behavior of shell structures. The turbulent spectrum of the wind provides its frequency composition. Pressure fluctuations are related to ambient velocity fluctuations.

Modern formula's and recent measurements are cited. The edge size where most of the wind energy is concentrated, is computed. Only the longitudinal velocity component is described, but the importance of the lateral and the vertical components is noted.

Constructions experience aerodynamic forces generated by the upstream flow and by the wake behind the structure. In addition, aeroelastic forces may be generated by the structure motion. A linear elastic MDOF system is simulated mathematically by means of a set of coupled differential equations: the first members have the three terms concerning mass, damping and stiffness respectively, while the second terms have the three terms concerning the turbulent forces in the oncoming flow, the forces in the wake and the forces due to aeroelastic effects repectively. These latter are assumed to be a linear combination of the displacement and its velocity.

The responses due to turbulence in the approaching flow and turbulence in the wake, respectively, and the aerodynamic damping are introduced.

It is noted that in the case of an open construction, such as a cooling tower, inside pressure coefficients exist and that these are essentially constant and negative, but can vary in magnitude depending on the circumstances.

Limits of validity of some formula's are introduced.

Positive aerodynamic damping, being conservative, is often "neglected" merely because the structural damping is not so well known either. Aerodynamic instabilities, such as galopping, are due to negative aerodynamic damping forces. The effect of turbulence is most noticeable in the across-wind direction.

Because the aerodynamics are not always easy to describe mathematically, a wind tunnel is often used. Herein the influence of nearby constructions or obstacles can be added. The simulated flow should be "appropriate". The main difficulty is modelling the length scale of turbulence, i.e. the energy frequency distribution. Even wind tunnel testing is not the answer to all problems. A good understanding of the forcing mechanisms is necessary prior to testing. The flow must be modelled accurately, certainly for relatively light and flexible structures.

Topics in the 19 references form quite a survey. Study cases concern open-topped oil storage tanks, cooling towers, full-scale observations, aeroelastic reponses, finite element random reponse analysis, earthquake, across-wind response of slender structures and response to vortex shedding.

3.4 **D.Van Gemert, C.Vanoverschelde, P.Wouters** and **J. Decock** present a study case, namely a concrete pedestal of the telecommunications antenna building at Lessive, Belgium. The vertical deflection of the roof center had to be smaller than 2mm and the lowest frequency, of the antenna building with soil, had to be greater than 2Hz.

Taking into account imposed esthetic requirements, the authors started with a nice concept and had to stiffen it gradually, adding interior columns and, finally, stiffening walls. The structure was idealized including foundation stiffness according to a formula taken from a book noted in the text. The thus idealized structure has 12 degrees of freedom.

The lowest natural frequency computed with SAP IV is 7.1Hz corresponding to an horizontal displacement, but experimental measuring of the Lessive III-Earth Station shows 3.2Hz as lowest frequency and this is in torsion. Two reasons for the discrepancy are given: in the analysis, all the connections are ideal, perfect fixations and the antenna was not included.

In fact, the imposed, severe requirements were well realized notwithstanding the idealization of the structure for the analysis.

3.5 **W. B.Krätzig** and **K.Meskouris** take into consideration that in most design codes, special provisions are included to ensure ductile performance of aseismic structures, thus requiring nonlinear analysis. A two-dimensional model is computed but the basic principles apply to 3D structural models.

The study case is a R/C frame of a six story building that was designed according to DIN 4149 and, separately, according to the CEB code.

As a first step for the evaluation of the nonlinear behavior of both design variants, static limit load analyses were conducted for horizontal loading for preliminary assessment of hinging patterns. The system ductility is lower for the DIN design, which also exhibits more unwanted hinge formation in columns.Nonlinear dynamic analysis (nonlinear time-history calculations) are performed for three spectrum compatible motions with durations between 6s and 8s, as well as a recorded accelerogram. The maximum reached curvature ductilities for all four motions and both design variants are compared with ductility supply (ductility balance method). Thus, no danger of collapse appears in any of the considered cases, but the weaker points can readily be improved.

The explicit ductility balance approach furnishes realistic insights in the structure's nonlinear behavior and thus focuses attention on its weak points. With the advent of micro-computers and thus cheaper computing power, such powerful analysis tools should soon find their way in everyday design practice

---

Reference topics: codes for seismic design of R/C structures, nonlinear seismic analysis of R/C structures, damage analysis of R/C buildings, computer program for inelastic response of R/C frames to earthquakes and nonlinear models for R/C frames.

3.6 **J.Eibl** and **L.Stempniewski** tackle dynamic analysis of liquid-filled tanks including plasticity and fluid interaction by earthquake effects,because it concerns a capacity of more than 100000m3 and highly explosive or otherwise dangerous fluids.

The available knowledge is reviewed and the Lagrangian approach is chosen. It describes the fluid behavior in terms of finite element nodes displacements which yield compatibility at the interface nodes between fluid and structure. The major advantage of this approach is that these elements can be readily incorporated in a general- purpose computer program.

The material nonlinearities and free surface sloshing are actually treated, while geometrical nonlinearity, soil-structure interaction including lift-up, are under work and thus not yet subject in this paper.

The isoparametric fluid element, the isoparametric shell element and the isoparametric contact element are defined and due references precised.

The resulting matrix equation of fluid-structure interaction excluding damping is considered. It comprises the inertial forces due to earthquake, the external pressure loads and the free surface displacements loads. Newmark's integration method is used. Equilibrium iterations are performed at each step because of the material-nonlinearity of the tank wall.

Unsymmetric pressure loading can lead to the "elephant footing" of the tank.

A tank of 20m diameter and 20m height is investigated. The nine seconds lasting N-S component of the Vranca earthquake (Bucharest) was taken. Material properties comprise E for the fluid and also its "hourglass"-module.

Results concerning the motion are listed.

The tank first plastifies at the top with resulting membrane hoop forces. The overturning moment tries to lift off the (anchored) tank on one tankside and initiates axial compression on the opposite side, initiating buckling or "elephant-footing".

The presented numerical tank model is applicable very well.

Topics in the references are the following: finite elements for fluid-structure interaction, transient interaction, "Explosionsdruck", dynamic analysis, containers, tanks.

3.7 **R.Motro, S.Najari** and **P.Jouanar** contribute to the use of tensegrity systems by a static and dynamic analysis of some systems. They remember the definition given by R.B.Fuller:"A tensegrity system is established when a set of discontinuous compressive components interact with a set of continuous tensile components to define a stable volume in space." The name tensegrity is derived from tension and integrity and designates very light structures combining compression and tension members to make systems which do not require the usual anchor blocks that go with tension structures.

The actual study is limited to rectilinear elements and nodes pertaining to one member in compression only. The prototype comprises nine cables and three struts.

The expounding comprises clearly stated conceptions such as the large displacement matrix, the initial stress matrix and the transfer function.

The five references, all in French, comprise: reticulated spatial selfconstrained systems; continuum mechanics related to structural analysis; automatism of linear systems; simultaneous iteration analysis to obtain eigenvalues and eigenvectors; identification of civil engineering structures by means of harmonic analysis.

4. Nonlinear analysis

4.1 **Yeong-Bin Yang** presents a general theory on the stability of thin-walled beams, the cross sections being allowed to warp nonuniformly along the axis of a member, such as in the classic theory by Vlassov.

In the large displacement analysis of a thin-walled bar, the equations of equilibrium are written, thus including instability effects. The torsional-flexural buckling of a bar with an arbitrary open cross section is analysed and practical applications in the case of a torque are presented.

The principle of virtual displacements is applied to the very deformed configuration, using the Cauchy stress tensor and the linear or Almansi strain tensor, but then a Lagrangian description introduces the second Piola-Kirchhoff stress tensor and the Green-Lagrangian strain tensor with the large-displacement strain-displacement relations.

The variation of the strain energy is expressed in terms of the axial, flexural and warping deformations. To that variation is added the contribution of the linear shear strains caused by the St.Venant torque. There are expressed the virtual work done by nonlinear axial strain and the associated applied loads potential in the variational form.

Finally is added the virtual work due to nonlinear shear strains, thus yielding the potential of the applied loads in the variational form. Thus are included terms neglected by foregoing authors.

Finally is established the full virtual work equation of equilibrium of a deformed thin-walled beam and, through integrating it by parts and admitting the arbitrary nature of virtual displacements, four nonlinear differential equations are obtained and general natural boundary conditions for each degree of freedom, at both ends of the beam as well. The thus obtained linearized differential equations, along with the natural boundary conditions, can be used as a valid basis for a large displacement analysis of thin-walled beams.

The theory is applied to the analysis of the torsional-flexural instability of bars with an applied torque, semitangential in one case and quasitangential in another. Results are listed and it is noted that for a circular shaft, the presented solutions coincide exactly with Ziegler's.

The fourteen references cover: finite element analysis of torsional-flexural stability; buckling strength of metal structures; space behavior of beam-columns; thin-walled bars; stiffness matrix for geometric nonlinear analysis; bimoment contribution to stability of thin-walled assemblages; principle of structural stability.

4.2 **M.A.Gizejowski** analyses plane frames sensitive to both geometric and inelastic material properties.

The presented nonlinear in-plane analysis of metal frames is able to consider any imperfection in computing a more correct value of the real structure failure load. The Newton-Raphson interpolation technique is used to follow the load-deflection history. This technique is combined with the gradient test to search for the limit point and to follow the full response till in the postlimit range. The concept of multi-pointed cross section is utilized to take into account the unloading effect of earlier plastified fibres and an arbitrary residual stress pattern.

The classical assumptions on beam behavior are made.

For any cross section, the initial configuration of the imperfect element and its intermediate configuration on the loading path are referred to the initial configuration of the perfect element. The sum of the corresponding displacements are differentiated with respect to the coordinate along the longitudinal axis of the perfect beam. The kinematics of an elementary beam part (limited by two cross sections) is expressed. Fully geometrically nonlinear behavior is incorporated.

The principle of virtual displacements yields the total equilibrium equation for the beam-column element. It is expressed that the residual strain is time-independant and self-equilibrated. The derivatives of the displacements are time-dependant. Natural boundary conditions and generalized element distributed loads are reviewed and splitted in linear and nonlinear components.

The full variational total equilibrium equation including all nonlinear terms is established.

The frame members are divided into finite elements with eight degrees of freedom in the points of their origin and end, i.e. the axial and transverse nodal displacements in the plane of the frame. The displacements and the initial geometric imperfections too, are approximated as a linear combination of the nodal values and four standard cubic shape functions, taken from large deformation in-plane analysis of elastic beams for the nondimensional isoparametric element. The introduction of these approximations in the variational total equilibrium, which is physically and geometrically nonlinear, is performed. A Taylor's serie expansion lets establish the element incremental equilibrium equations and only one nonlinear term in increments is retained. The approximate element incremental equilibrium equation is finally written in matrix form. The coefficient matrix is called tangential (or incremental) stiffness matrix. The unknowns are the displacements defined in the coordinate system of the ideal form of the beam. The force vector is called element unbalanced (or incremental) nodal forces. The stiffness matrix is a sum of the following main matrices: the constitutive stiffness matrix, the geometrical stiffness matrix and the surface traction matrix. The nonlinear component of the first is called the initial displacement stiffness matrix and the nonlinear component of the second is called the initial stress stiffness matrix.

To take sufficiently into consideration the influence of initial residual stresses and the strain history (the unloading effect of earlier plastified fibres and arbitrary residual stress patterns; stress-strain diagram is irreversible) the one-layer multi-point cross section conception is used. All flange and web cross section components are discretized by dividing into finite elements. The average values at the centroids are computed and are contributions to the Gauss point values. Weighted sums yield the matrices and column matrices of a beam element.

The obtained values are transformed to the global frame coordinates for assembling, performing equilibrium and compatibility in the nodes.

The total displacements of the structure nodes are obtained by simple addition of the increments.

Tested examples showed that the method allowed for all kinds of nonlinearities, is effective from the practical point of view because of its relatively small computer CPU time consuming. The computer program was implemented on a PDP computer.

The seven references cover one general book, proceedings of a conference (nonlinear analysis of frames sensitive to the instability effects)) and five articles concerning instability of rigid-jointed frameworks, large displacement analysis of 3D beam structures, 3D FEM for elasto-plastic frames, nonlinear FEM for steel frames.

4.3 **Y.Hangai** presents an analytical method for tackling the vicinity of critical points.

After an introductory survey of the methods of computation of nonlinear equilibrium paths with limit points and bifurcation points in the cases of multi-degree-of- freedom systems, the definition of the generalized inverse is introduced and applied to the solution of systems of linear equations with rectangular mxn matrix $\underline{A}$. The generalized inverse is an nxm matrix satisfying four relations to $\underline{A}$. For m=n, the generalized inverse is simply the inverse of $\underline{A}$.

The generalized inverse $\underline{A}^-$ of $\underline{A}$ permits to formulate an existence condition of solution. The system of equations being

$\underline{A}\ \underline{x} = \underline{g}$ ($\underline{A}$: mxn matrix; $\underline{x}$: nx1 matrix; $\underline{g}$: mx1 matrix),

the existence condition is

$\underline{A}\ \underline{A}^-\ \underline{g} = \underline{g}$

and a solution may be

$\underline{x} = \underline{A}^-\ \underline{g} + (\underline{I} - \underline{A}^-\underline{A})\underline{\alpha}$

($\underline{I}$ is the nxn identity matrix and $\underline{\alpha}$ any nx1 vector)

The column vectors of ($\underline{I}$ - $\underline{A}^-\underline{A}$) being denoted by $\underline{a}_1$, $\underline{a}_2$, ... $\underline{a}_n$, the solution becomes, when p is the rank of ($\underline{I}$ - $\underline{A}^-\underline{A}$),

$\underline{x} = \underline{A}^-\ \underline{g} + \alpha_1\underline{a}_1 + \alpha_2\underline{a}_2 + \dots + \alpha_p\underline{a}_p$ (α_i is any scalar)

with $p = n - r$

wherein r is the rank of A.

A discrete system is described by n displacement parameters D_i and a load parameter Λ. The system of nonlinear equilibrium equations

$f_r(D_i,\Lambda) = 0$; $(i = 1, \dots, n)$; $(r = 1, \dots, n)$,

is obtained by any discretizing method from the continuous system.

A known point of the equilibrium path is (D_i^o,Λ^o).

A neighbouring point is $(D_i = D_i^o + d_i;\ \Lambda=\Lambda^o+\lambda)$, such that the relation

$f_r(D_i^o+ d_i,\Lambda^o+ \lambda) - f_r(D_i^o,\Lambda^o) = 0$

yields, after a Taylor's expansion, the localized nonlinear incremental equilibrium equations in terms of d_i and λ for the known point (D_i^o, Λ^o).

The increments d_i and λ are functions of one parameter t and are accordingly expanded in McLaurin series. These, limited to the square terms, are introduced in the incremental equilibrium equations. The equating of the coefficients of equal power of t yields the incremental equations from which starts the classification of critical points.

These equations are

$\underline{K}\dot{\underline{d}} + \underline{h}\,\dot{\lambda} = \underline{0}$ (1st incremental equation)

$\underline{K}\ddot{\underline{d}} + \underline{h}\,\ddot{\lambda} + \underline{r}(\dot{\underline{d}},\dot{\lambda}) = \underline{0}$ (2nd incremental equation)

$\underline{h}$ is the load and $\dot{\lambda}$ the derivative of the increment λ of the load factor Λ; the components of the vector $\underline{r}$ are defined.

When $\underline{K}=\underline{0}$ the point (D^i, Λ^o) is a critical one.

The existence condition of the 1st incremental equation is

$(\underline{I} - \underline{KK}^-)\underline{h} = 0$

When n-1 is the rank of $\underline{K}$ the rank of $(\underline{I} - \underline{KK}^-)$ is $n-(n-1)=1$ and this matrix has then but one linearly independent column vector, say $\underline{k}$.

The solution of the first incremental equation is then

$\dot{\underline{d}} = \underline{K}^-\underline{h}\,\dot{\lambda} + (\underline{I} - \underline{K}^-\underline{K})\dot{\underline{\alpha}} = \underline{K}^-\underline{h}\,\dot{\lambda} + \alpha_1\underline{k}$ (α_1 = any scalar)

while the existence condition takes the form

$\underline{k}'\underline{h}\,\dot{\lambda} = 0$ ($\underline{k}'$ =transpose of $\underline{k}$).

With $\dot{\lambda} = 0$ and $(\underline{k}'\underline{h}) \neq 0$ the critical point is a limit point and the solution becomes $\dot{\underline{d}} = \dot{\alpha}_1\underline{k}$, thus with a buckling mode $\underline{k}$.

When $(\underline{k}'\underline{h})=0$, the author proves that

$\dot{\underline{d}} = \underline{k}\dot{\alpha}_1+(\underline{K}^-\underline{h})\dot{\alpha}_2$ with $\dot{\lambda} = \dot{\alpha}_2$ ($\dot{\alpha}_1$ and $\dot{\alpha}_2$ arbitrary scalars)

which means two distinct equilibrium paths emerging from the critical point, which is then called a bifurcation point.

The existence condition of the second incremental equation leads the author finally to a quadratic equation of $\dot{\alpha}_1$ and $\dot{\alpha}_2$ thus yielding two ratios $\dot{\alpha}_1/\dot{\alpha}_2$.

Thus it is possible to pursue numerically the two post-bifurcation paths by means of an analytical method which is based on the use of the generalized inverse.

The possibilities of the presented method comprise the fact that the selection of the incremental parameters is not necessary.

Amongst the topics in the references are: shallow circular arches, space frames, snap-through buckling of reticulated shells, generalized inverse and geometrically nonlinearity, matrices in statistics, instabilities and catastrophes in science and engineering.

4.4 **M.Van Laethem** and **J.de Coen** apply the computer program TITUS to a beamless barrel vault with two curvilinear prestressing cables. It spans 18m, has a central angle of 68.4°, a radius of curvature of 11.56m and 7cm thickness.

The triangular shell plate element gives the best results and is the one available for nonlinear computations.

Nonlinear computations are applied because the displacement of the central edge points equals nearly the shell thickness, what is contrary to a basic assumption in linear shell theory.

As pointed out by Kvetoslav Zahradnik, all the centers of gravity deflect like in any beam but the shear force has a big influence. The self weight causes the deflection of the center of gravity of the initial cross section to be 1.87cm. The nonlinear computation yields 1.55cm.

Under the effect of prestress only, the comparison of nonlinear to linear computation lets conclude that the nonlinear yields more severe results: the edges diplacement in their midpoint is 5cm instead of 2.8cm; a transversal bending moment of -1.3kNm/m instead of -0.75; a normal stress resultant of -10kN/cm instead of -8.5; a longitudinal bending moment of nearly -4kNm/m instead of -2.5.

Because the values due to the own weight have a different sign, and because the prestress is introduced before the own weight is allowed to work fully, a realistic nonlinear computation should simulate the real process, as said during the presentation of the paper. The effect of the nonlinearity is thus reduced.

Topics mentioned in the references are mainly investigations on the considered barrel vault and the origin of the computer program used.

4.5 **M.Cervera, A.J.Kant** and **E.Hinton** present a finite element for simulating the behavior of concrete structures accusing combined geometric and material nonlinearity. Examples involve also snap-through and/or snap-back behavior.

When the deflections are of the same order as the shell thickness, geometric nonlinearity is attained, yielding buckling and bifurcation.

The proposed numerical model features are: allowance of large displacements and rotations, concrete tensile cracking and compressive yielding and crushing, steel yielding and an efficient equation solution procedure able to megociate limit points.

The finite element comprises transverse shear effect and is an isoparametric degenerated shell element. The vector of nodal degrees of freedom consists of three orthogonal midsurface displacements and two independent rotations component.

The volume integrals appearing in the discretized equilibrium equations are evaluated using product integration rules resulting from Gaussian quadratures for area integration and Gaussian or Lobatto rules for the through-thickness integration. Shear and membrane locking, zero energy modes and failure to pass the patch test can necessitate special techniques. The selective integration gives reasonable results in the case of moderately thin concrete shell structures.

Increments of second Piola-Kirchhoff stresses are related to Green strains in biaxial conditions taking into account yielding surfaces, failure envelopes and hardening curves.

A strain-softening curve is introduced. In order to overcome the problem of mesh independency, the present work adopts a strain-softening relation based on the fracture energy concept, which may be obtained as a fundamental material property.

The reinforced bars are idealised by a smeared layer with uniaxial properties.

An arc-length solution procedure based on a spherical path is implemented in conjunction with a modified Newton-Raphson incremental-iterative procedure, with an automatic load incrementation scheme. Thus snap-through and snap-back can be handled, the convergence charateristics are improved and self-adaptive load incrementation is possible.

The load factor incrementation depends upon a convergence path constrained to follow a spherical path in the n-dimensional displacement space. The sign of the local increments has to be changed when the sign of the determinant of the tangential stiffness matrix changes. The "grading" of the solution procedure in the areas of most rapid change can be accelerated.

Four numerical applications are presented.

For a thin cylindrical shell, the load-deflection curve, with two limit points and a vertical tangent in between, matches perfectly a curve published by Sabir and Lock. A figure shows a beamless small model of a barrel vault, but the global angle at centre is only 0.2 rads, the radius is 2540mm,the span width 254mm, the thickness 6.35mm. The material characteristics are about E=3.1kNmm-2 and
ν=0.3. A central point load roughly reaches 0.6kN, falls to
-0.4 and raises finally till 0.75kN while the displacement is successively about 12mm, 18mm and finally 30mm. The mesh for the whole shell shows 16 elements. A quarter of the shell was considered.

The second example treats a reinforced concrete hypar on a square base and supported in two corners, which behaves almost linear, up until failure. Two computational methods are compared. The first is based on a local softening curve with an ultimate tensile strain of 0.00015. The ultimate load obtained compares well with a value published by

G. Mueller for the same model. The second computational method is based on the fracture energy of 0.25N/mm. A much lower failure is then obtained, thus showing the need for an objective method of defining the softening branch of the tensile model. The finite element model of a quarter shell consists of 14 Heterosis elements: 10 to represent the shell and 4 to represent the edge beam.

The third example concerns a reinforced concrete rectangular plate of 244cm x 122cm x 3.2cm. It is fully clamped and compressed in the direction of its length and supports a small transverse load to initiate the buckling process. As known from two publications, reporting tests, the plate buckles with one half wave in each direction. Only a quarter of the plate is modelled, i.e. with six shell elements. The results compare better with the experimental results than those obtained with an elsewhere published displacement incrementation analysis. A phase difference with the experimental results is only due to the difficulty in obtaining a true clamped boundary condition in practice.

It is concluded mainly that the proposed numerical method lets analyse concrete shell structures which exhibit both geometric and material nonlinearity.

The topics in the 14 references are total Lagrange formulation, transient dynamic analysis of reinforced concrete structures, finite elements, ultimate load analysis, buckling, fast computation of snap-through, tracing nonlinear responses near limit points, arc-length solution techniques, large deflection, numerical problems, tests, etc.

4.6 **J.Spiekhout, A.M.Gresnigt** and **G.M.A.Kusters** adapt a calculation model to experimental results in order to predict the real local load-indentation depth diagram for a steel cylinder under the influence of a local load in the elastic and elastoplastic area.

The computer programma DIANA of the IBBC-TNO was used, taking into account elasto-plasticity, large displacements and non-conservative gas pressure.

Indentation in pipe lines is caused by a hydraulic excavator. Then the burst pressure is considered.

First, formulae are considered and then finite element computations.

The formulae for the dent depth caused by a local force is given first in the area of elastic deformation and then in the area of plastic deformation.The dent depth is substantially smaller in the presence of an internal pressure. The formula for the elastic range is based on the shell theory of a pipe loaded with two diametrically opposite forces. This formula is then modified by replacing a value by its square root in order to match the test results. In the plastic situation, four hinges are determined from experience, and equilibriun of a shell part is expressed. The contribution of the internal pressure was derived by means of virtual work. To match reality, a coefficient is adapted.

The finite element analysis matches closely. The considered computer program comprises ten types of finite elements, from solid till reinforcement elements. Also nonlinear springs are present and served to simulate soil pressure. An example of an element mesh of a quarter pipe is shown, also the calculated and the experimental load-displacement curves, once for zero internal pressure and once for internal pressure, which shows the good influence of this pressure. Also the corresponding indentation patterns are shown.

In the conclusion is reported that the results of the finite element calculation as well as the test results were used to adapt the considered formula. The finite element calculations provided a good insight into the influence of the various parameters on the indentation depth. For instance, support conditions had only little influence.

The references comprise topics as follow: local charges on cylindrical shells and on spherical shells; pipe connected to a spherical shell; computer program DIANA on a micro computer; mechanical damage of pipelines.

4.7 **Sun Jingyu, Anders Peterson** and **Hans Peterson** use a PULD formulation for nonlinear analysis of thin shells, strains remaining small but the displacements becoming eventually large.

PULD means partially updated Lagrangian description. The reference configuration coincides with a translated and rotated region of the undeformed body. (It is recalled that in a referential description the independent variables are the position of a particle in a chosen reference configuration and the time. In total Lagrangian description and updated Lagrangian description respectively, the initial undeformed and the current stressed configurations are chosen as the reference configurations.)

An undeformed element subregion is taken as reference. The integration over an undeformed is easier than over a deformed. In order to match the result of the latter, an extra term is added to the first, without loss of accuracy. Thus the PULD formulation may be considered as a modified or "approximate" updated Lagrangian formulation.

The general FEM formulation of nonlinear problems is exposed in tensor notations. The second Piola-Kirchhoff stress tensor increment is related to the incremental Green strain tensor by means of a symmetrical material tensor.

The material behavior is assumed to be isotropic and linear elastic. The Galerkin method is applied with the displacements taken as weight functions. The element stiffness matrix is the sum of one linear part and two nonlinear contributions, i.e. the initial stress stiffness and the initial displacement stiffness.

Two kinds of elements are used. Both have the shape of a space quadrilateral subdivided into four triangular elements. These triangles are TS1 (12 d.o.f.) for the one kind and TS2 (27 d.o.f.) for in the second. The stiffness and the load matrices of the quadrilateral elements are formed through transformation, assembly and condensation. The derived finite element equations for shell analysis have been implemented in a subset of the CAMFEM-system (Computer Aided Modelling based on the Finite Element Method)

Some numerical examples verify the equations and the computer program.

A cantilever plate beam under concentrated loads at its free end deflects accordingly to an elliptic integral solution. The result of the finite element analysis taking the initial displacement stiffness matrix into account is much closer to that exact solution as is the result obtained without this matrix.

A spherical shell (R=2.54m, thickness=0.09945m, $E=68.95 \cdot 106 N/m^2$, $\nu=0.3$) covers a square basis (side=1.5698m) and is subjected to a point force P at its crown. All edges are hinged and immovable. Only one quarter of the shell is analyzed by using one element only for TS1 and TS2. The latter only matches earlier published results. In both cases, the use of the initial displacement stiffness matrix improves the results. The first limit point is reached for P equalling nearly 50kN and the deflection ...130... 140...mm. Then follows what should yield the snap-through till about 270mm followed by an increasing stiffness.

A cylindrical shell (Radius=2.54m, length=0.508m, central angle=0.2 radian, thickness=0.0127m, $E=3.10275 \cdot 109 N/m^2$, $\nu=0.3$) is subjected to a point force at the midpoint of the upper straight line. The curved edges are free whereas the horizontal boundaries are hinged and immovable. The shell is analyzed by using TS1 and a 4 x 4 mesh for one quarter of the shell.The load-deflection path matches well previously published results. The first limit point corresponds to a load of about 2.2kN and a deflection of about 12mm. Then the snap-through should reach 27mm.

It is concluded that PULD has the advantage over ULD (Updated Lagrangian Description) in shell analysis. The influence of local element deformations is taken into account.

Nineteen references are listed. We note the following topics: finite elements for large deformation dynamic analysis; total and updated Lagrangian descriptions in nonlinear analysis; geometric and material nonlinearity; finite element procedures; co-rotational finite element formulation; mechanics of continuous medium; high temperatures; nonlinear transient finite elements with convected coordinates; plates; CAMFEM; large deflection beam and frame problems by means of elliptic integrals; finite deformations of shallow shells; tensor notations for nonlinear finite shell elements; triangular thin shell finite element for nonlinear analysis; finite element instability analysis of free form shells; nonlinear shell analysis using free formulation.

4.8 **Arata Yoshida, Shigeru Ban, Haruji Tsubota** and **Kazuo Kurihara** present a new type of tensile fabric structure and its nonlinear analysis. This structure covers about 30m x 30m free space and consists of steel arch frames, fabric panels and steel cables. At first the fabric panel is prestressed by means of clamping nuts and bolts. Secondly, the cables, which are attached to the centre of the fabric panel, are prestressed. For the first, the equilibrium shape is calculated. Then, when the cable tensile forces are induced, the caused stress and displacement state is calculated. This state is considered as the initial shape. Finally the snow or wind loads are applied and the consequent state calculated.

An experiment on a panel at $\frac{1}{2}$ scale was performed.

The principle of the form finding analysis is to assume a state of stress corresponding to a given load and to express the equilibrium by means of equations in which direction cosines are functions of unknown coordinates. Triangular membrane elements converging at a node constitute the basic unit. Thus equations are expressed for each node in order to obtain a system of nonlinear equations, which is solved by Newton-Raphson technique. Applying Taylor's expansion with respect to coordinate increments and neglecting their second order terms, a linearized system is obtained, which tangential stiffness matrix is established.

The large deformations caused by snow or wind are analysed by the finite element load incremental method using updated Lagrangian description. The fabric is assumed to be elastic but anisotropic. The Cauchy stress matrix is considered.

The tested panel method at the $\frac{1}{2}$ scale covered a square base of 4m side and was clamped at edge arches with 0.4m rise. At the panel center, a rigid plate was connected to diagonal cables. The two kinds of prestress were introduced and measured. The first yielded 50N/cm in the fabric and the second a 2kN vertical component at the panel center. Distributed loads were then simulated by 25 equivalent concentrated loads: 1.2kN/m^2 downward for snow and 3.5kN/m^2 upward for wind.

External loads are sustained mainly by warp fibers which are stiffer than fill fibers: this is seen from calculations and from experiment.

The cables are efficient against wind load but loose their tension under snow load.

The load-displacement curve of the center is linear and calculation matches experiment. For a point about half way between the center and an edge, the experimental curve shows smaller displacement than the calculated curve when the displacement attains 100mm and more, but the discrepancy is acceptable. However, a more accurate analytical model should take the nonlinearity of membrane material into account.

A photo shows the overall view of the actual tensile fabric structure, which was completed in March 1986 in Tokyo.

Five references close the contribution. Following topics are covered: large-span pneumatic structures reinforced by steel cables; finite element formulation for large deformation dynamic analysis; CAD and structural design of cable reinforced membrane structures; biaxial deformation property of coated plain fabrics.

4.9 **D.H.Jiang, M.C.Chang, C.J.Hsiao, C.C.Lee,Y. F.** Lee and **W.P.Lin** apply nonlinear analysis to buried pipelines and compare the results to results of tests on an actual pipe in the laboratory.

Poor subsoil and traffic let expect unequal settlement.

Reinforced concrete is the considered material. When the thickness of cover protection ($\frac{1}{2}$ in. minimum if exposed to weather and 3/8 in. minimum if not) is discounted, the wall thickness is less than one tenth of the radius such that, according to Roark, the membrane stresses caused by uniform pressure are practically uniform throughout the thickness of the pipe.

The formula of Timoshenko and Gere for a two point loaded ring express the ovalic displacement in the elastic range. This expression eliminates the radial displacement in the formula of the hoop or ring bending moment. This bending moment reaches his maximum in the elastic range when the yield stress is reached.

Then is considered Donnell's formula of the critical stress in moderately long cylinders subjected to hydrostatic loading. However, for design purposes, a conservative curve is selected at $E/f_c'=667$. This is introduced, with $\nu=0.18$ in Donnell's formula. The formula of Lame serves then to express the cylinder wall stress in the hoop direction in function of the uniform pressure. The stress herein is replaced by the value taken from Donnell's formula and thus is obtained a critical pressure. However, a factor is introduced: its value is 1 for simple ends and 1.2 for fixed ends.

The diameter of the tested pipes ranges from 200m till 500mm, but only results of the 350mm diameter pipes are reported. The ovalic displacement according to the cited formula of Timoshenko and Gere is matched well by the test results. The buckling load found by the tests is 170N/cm^2 while the calculated critical pressure is 189N/cm^2.

In the conclusion is reported that the values in the case of fixed-end supports are advantageous when compared to the case of line supports such that on weak soil, pile foundation under the pipe connection not only serves to prevent leakage but also to strenghten the pipe. Buckling occurs at large curvature in the longitudinal direction and after the measured internal pressure is higher than the limit for yielding: buckling thus occurs after the pipe has become partially plastic. The visual fissure began right after the yielding point and the crack happened on the ultimate strength loading. Nonlinearities due to plasticity, time-dependent effects and loading rate are still a target for further study, but a well trained engineering judgment can be a good help for those merely applying the existing formulas.

In the references, particular topics close to the subject are: plastic design of buried steel pipelines; buckling of shells and shell-like structures; flexible shells; plasticity in reinforced concrete; theory, experiment and design of thin-shell structures.

[1] **Shell and Spatial Structures : Computational Aspects**
Eds.: G. De Roeck, A. Samartin Quiroga, M.A.V.A. Van Laethem, E. Backx; **Springer-Verlag**; 1987. Approx.495 pp. Soft cover DM 66,-. ISBN 3-350-17498-2. Springer for Science, P.O.B. 503, 1970 AM IJmuiden, The Netherlands.

4.1 **M.K.Nygård** and **P.Bergan** introduce an unconventional class of elements for nonlinear shell analysis.

The excellent performance of various types of linear membrane and plate elements obtained previously by the author by means of their free formulation theory, has incited them to apply this theory to nonlinear problems with large displacements and inelastic materials.
also thin shell elements are elaborated by use of the free formulation theory with the introduction of "drilling freedoms", i.e. rotational freedoms normal to the element plane.

The free formulation theory is explained at first for linear formulation. The displacement field is put as a set of linearly independent displacement modes: one part is a complete set of rigid body modes and constant strain modes; the second part is a set of higher order displacements modes. A standard finite element formulation would then give an element that satisfies completeness but seldom the interelement compatibility,such that it would not converge. The author's alternative is to form the element stiffness by the following two contributions: at first, the basic stiffness, i.e. the contribution of the constant strain/stress behavior; secondly, the higher order stiffness. Such an element stiffness passes the patch test even for incompatible displacement functions.The considered relaxation of constraints brings in a new class of finite elements, particularly useful for C^1 type of elements, such as plate and shell elements.

The free formulation theory may be extended to situations with displacements and strains of arbitrary magnitude and inelastic behavior with strain history.

The plate and shell elements are initially flat and the usual Kirchhoff thin plate assumptions adopted. The rotational freedom of any node about the normal to the plane of the plate is called drilling freedom and is defined as the mechanical rotation derived from the in-plane displacement gradients.

For the triangular plate/shell element the membrane action comprises the in-plane "bending" modes oriented along the median lines of the triangle; the bending action is expressed in terms of area coordinates. In both actions, three rigid body, three constant strains (or curvature) and three higher order modes are put. The integration is exposed.

For the quadrilateral element, membrane action and bending action are considered and also the special integration features. Nondimensional coordinates are used. This is also the case in the triangular element where also the same rc-modes (rigid-body and constant strain) are set as in the considered quadrilateral element for the membrane action. The bending action is composed by the rc-modes for the out-of-place displacement (the six terms of a complete second degree polynomial). For the higher order bending modes, six functions of the two nondimensional coordinates are chosen. Therein are three constants, to be determined from the energy orthogonal requirement.

The analysis of shells subject to large displacements (and large strains) takes advantage of the "gost reference description", i.e. a Lagrangian description (referring to the initial configuration of each element) but with the reference configuration closely displaced and rotated along with the deformed configuration. Also, a new procedure for treating large rotations in space is applied.

The computer program FENRIS is part of the SESAM system, comprising interactive pre- and postprocessors, comprises FTR and FFTR (free formulation triangular shell elements) and FFQ (free formulation quadrangular shell element) and is applied in four numerical examples.

The shear loaded cantilever beam is computed with far more efficiency with the considered elements than with the constant strain triangle or with the linear four-noded displacement based quadrilateral.

The cantilever beam subjected to end moment is nearly perfectly bend 360 degrees after 20 load steps.

The cylindrical shell roof suggested by Scordelis and Lo has become a classical problem for testing shell elements. The considered finite elements (FFTR and FFQ) give better results than the three node TRUMP element by Argyris et al. and the four node element by Batge and Dvorkin.

A cylindrical shell hinged along its straight edges and free along its curved edges is subjected to a concentrated load. Displacements computed are in good agreement with other results reported in the literature. For the snap-through behavior, the FFQ yields the best among other good results.

The paramount conclusion is that the free formulation theory for finite elements with in-plane bending modes, associated with membrane type rotational ("drilling") freedoms, improves the accuracy significantly.

Twenty-three references cover finite element formulations, patch test, rotational degrees of freedom, shear deformations, energy orthogonal functions, shells, nonlinear analysis, general purpose nonlinear finite element program, description of SESAM, elasto-plastic and elasto-viscoplastic materials, mixed interpolation of tensorial components, total Lagrangian formulation, hyperplane displacement control methods in nonlinear analysis.

4.11 **P.Morelle** and **G.Fonder** treat shakedown and limit analysis of shells by means of a variational and numerical approach.

A linear distribution of the strain along the "normal" (linear kinematic hypothesis, i.e. Kirchhoff-Love hypothesis) does not automatically generate a linear variation of the stresses along that normal in the case of an elasto-plastic material. So, the set of generalized strains and stresses has to be completed with "pseudo-residual stresses".

The residual quantities are defined as the differences between the elasto-plastic time dependent and the elastic solutions. The residual stress field is decomposed in two terms: the first is the linear part; the second yields residual vanishing stress and is thus called "pseudo-residual stress". Thus, the linear part of the residual stress field only, yields the residual generalized stress.

The plastic behavior of shells is described in terms of generalized residual stresses. (The limit analysis tackles the rigid plastic behavior, in which the time derivative of the elastic strains is practically vanishing. This is a common approach disregarding the "Prager reactions"). The decomposition of the total stresses in the three considered terms is introduced in the constitutive equations, which lead to the determination of a hypersurface in the space of generalized stresses. This hypersurface is called elastic locus.

The variational study of Hencky and Kirchhoff plates is based on the variational principle which states that the variation of the time derivative of the elastic locus vanishes. The conclusion of this study is that the pseudo-residual stresses have not to satisfy any "volume" equilibrium equation but only the condition that they yield vanishing generalized stresses.

The adaptation of shakedown of plates and shells happens when the time derivative of the strain tensor throughout the volume is zepo. This leads to the condition of shakedown in terms of generalized stresses: the time derivative of the residual generalized stresses must vanish and also the time derivative of the pseudo-residual stresses.

A theorem of Melan leads to a first theorem, which leads to the conclusion that in the generalized stress problems, shakedown analysis requires the construction of residual generalized stress fields and an appropriate family of elastic loci, which altogether means a good choice of pseudo-residual stresses.

An automatic method of determination of these stresses is introduced, based on the use of finite elements and mathematical programming methods. The elastic generalized stresses, the residual generalized stresses and also the pseudo-residual stresses are discretized, the latter in layers.

A load domain depending on shakedown factors is considered. The shakedown problem consists in finding the biggest shakedown factor which guarantees adaptation of the structure for any variations of the loads within the considered load domain.Thus a second theorem is expressed, according to which it suffices to check satisfation of a plastic condition for the vertices of the load domain.

The limit analysis is specified as a particular case of shakedown.

The static shakedown analysis is specified as a particular case of shakedown.

The static shakedown formulation for plates and shells is introduced as a generalisation of the J.A. König's method for frames and arches. Use is made of statically admissible elastic finite elements to compute the discretized generalized elastic stresses. A similar discretization of the generalized residual stresses is also performed. Then a special form of discretization of the pseudo-residual stresses starts from a subdivision in layers and is based on the expression of the plastic condition. Adding the equilibrium conditions for the generalized residual stresses, finally yields a mathematical programming model.

The shakedown problem formulation is finally expressed and applied successively to a beam, a portal frame and axisymmetric shells. It appears that a linear or a linearized yield surface is to be preferred.

The applications are in fact under development and numerous examples particularly in the field of axisymmetric thin shells are expected in the near future.

The authors have indeed presented a general formulation allowing to solve shakedown or limit analysis problems in terms of generalized variables.

Correct results are expected even for partial collapse over the cross section,i.e. for low-cycle fatigue at points located within the shells, thus cases not properly detected by the classical "sandwich" formulation.

The references cover topics such as variational methods in elasticity and plasticity, shakedown of elasto-plastic structures, ACDPAC (a routine to solve differentiable mathematical programs), plasticity fundamentals, plastic shakedown analysis of plates and shells of revolution by equilibrium finite elements and optimal linearization of the criterium of von Mises.

In appendix, the variational study of elasto-plastic Hencky plates is exposed.

5. Presentation and interpretation of results

5.1 **A. Samartin** and **J. Cordona** introduce an algorithm for graphical computer results in shells following the principle of finite element technique to interpolate. However, the condition for the interpolating function to be a polynomial
is replaced by a mimimizing condition of a given smoothing functional.Interpolation techniques apply in graphical output of F.E. analysis (stress contour, lines of principal stress etc.), topography, design of cars, ships, etc. with given conditions of continuity, slope and curvature. Lagrange, Newton interpolation, splines for simple cases are well known. The capability of the F.E. method in this matter of interpolation is not completely exploited yet and the authors apply it, using important contributions to splines published already.

The general interpolation problem concerns a bounded two-dimensional domain wherein the searched function has to belong to a given space of functions and has to match values of derivatives in a number of given points. The set of derivatives in these points is not necessarily complete (up to a given order) nor consecutive. Besides, the function must minimize a functional which is the integral (over the domain) of a sum of the squares of the function and its derivatives, all those terms being affected by a constant, unknown coefficient.

The finite element method is used to describe an approximation for the searched function. Local discontinuities are taken into account by means of "hinges". This is very useful if Green functions (influence lines) are to be interpolated. The efficiency of the introduced interpolation method is shown for a function corresponding to a spherical surface.

The mesh used to obtain the results of a F.E. analysis can be used again in the interpolation procedure. In this way the input data for the list case is reduced to a minimum continuity: requirements of the graphical output of the results can be easily fulfilled by increasing the order of the interpolation functional.

The ten references concern the following topics: "finite elements and approximation"; numerical methods; fitting curves; splines and interpolation; finite elements of class C^m (in Spanish).

5.2 **J.P.Rammant, E. Backx** and **L. Knoops** developed finite element programs for the analysis of shell structures directly for a microcomputer. The use is at a fraction of the cost of a mainframe.

The program structure is shown in a block diagram, from which is considered the shell part. Other parts can be chosen as well from the main menu (choice of analysis) from which "radiates" [2-D beam], [3-D beam], [Shell], [Plate], [Volume], etc.. [Shell] joins then a five parts block: [Data Entry], [Data Verify], [Computation], [Report] and [Post-Processing]. A paramount feature is the [Data Editor], joined to the [Data Entry] together with a [Project Libr.] and an optional [Preprocessor]. The [Data Editor] is user's friendly and made up as follows: all data items (coordinates, topology, materials, ...) are added in sequence. Other advantages are exposed.

The elements used in the shell program are : quadrilateral shell element; 3-D beam element. The first is exposed.

During postprocessing, the user can define new combinations of load cases. The number of nodes, elements and load cases has virtually no limit for it solely depends on disk space thanks to the use of a data base manager.

In dynamics, the following are possible: eigenvalue/eigenvector extraction; time integration using modal analysis; spectral analysis; time integration and base acceleration.

Amongst the advantages exposed, the user's friendliness of the "Project library" amongst other items as "Report writing", "Graphics and CAD coupling", is noted.

The shell program, amongst others, was written to be used by engineers such that no system commands have to be renumbered to work with it.

Three references are listed: development of two simple shell elements; SAPIV; a non-conforming element for stress analysis.

5.3 **J. F.Stelzer** uses microcomputers for the interactive generating of spatial meshes and results evaluation.

Microcomputers working with an interpreter language offer, with their interactive possibilities, a superior quality in generating meshes for finite element analysis and effective graphics. Results interpretation is considered specially.

The routines are organized userfriendly and comfortable.

A menu presents numerous topics amongst which: checking the data; calling the automatic mesh generator, input of boundary conditions (restraints), graphical mesh interpretation, front width minimization, mesh transformation, joining of two or more meshes, adding or removing of nodes or elements. The points related with computer graphics only, are exposed.

Automatic mesh generation starts by the input by hand of the topology of one or more giant elements, i.e. parts of the whole structure volume. The subdivision follows all specified shapes of those parts. An example is shown.

Coordinate transformation and joining of two meshes is also shown by an example. The program asks for the nodal points of the separate meshes which must be combined to one and renumbers the nodes and elements.

Using different coordinate systems is for example applied to the combination of a torus part and a tube. A rough mesh is used, but refined after the transformation of the appropriate coordinates of each surface into the cartesian space.

Graphics representation of 3-D structures to represent the meshes generated in the preprocessor cover: wire frame presentation (curved edges being drawn with curved lines); hidden line removal; hidden surface removal (method by Stelzer) leading even to photograph-alike shaded images ressembling to photographs of structures not yet existing in reality); surface colouring to show regions of different materials; displaying subgroups (as example is shown a hidden line plot of a tractor lever surrounded by its subgroups in the shaded hidded surface plot mode).

The postprocessor for the evaluation of results permits the wise application of results graphics as the up-to-day way to show impressively what is going on in a structure. In the presented software the following possibilities are offered: 1) wire frame pictures - a) printing the result values at the nodes; b) deformation plot (poor); 2) hidden line display - a) deformation plot; b) with contour lines on the remaining visible surfaces (good); 3) hidden surface representation - a) deformed structure in the shaded mode; b) structure with coloured contourlines on the visible surfaces; c) structure (deformed or undeformed) with smooth colouring of the visible surfaces showing the function values with the spectrum between blue (lowest value) and red (highest value) (excellent). The last is detailed in an application to a part of a lever housing a bearing: a part is removed to show better the internal of the bearing, i.e. the area where the highest stresses are to be expected. The stress optics and the thermography are in a certain way leaved behind.

The technique of colour graphics is exposed. As a rule to day, a driver is still necessary to take the pixel information of the screen and change it in a way the hard copy understands and, furthermore, converts the additive in substractive colours. The driver presented, couples HP computers of the series 9000 with Tektronix hardcopies.

The application to shell structures happens without difficulties: a complicated shell construction loaded by the external atmospheric pressure, i.e. a thin-walled vacuum vessel, is shown by means of a shaded structure plot on the one hand and a deformation plot on the other.

The described capabilities are part of FEMFAM, a finite element package for big jobs on small computers.

Five references cover strategies in developing finite element software for desktop computers, hidden line algorithm, graphical postprocessing of finite elements results on microcomputers, colour FEM results presentations, "colour dump" of PROFEM (D-5100 Aachen).

5.4 **G. De Roeck** and **C. Vanoverschelde** link a powerful existing CAD-system for pre- and postprocessing of FEM calculations. The integrated database, allows fast development of processing, has more interchange possibilities than even general purpose pre- and postprocessor programs for FEM applications, and a far better way to expand the horizontal integration, i.e. in general some modules for manufacturing purposes.

An analogous improvement would be to use graphical software avoiding also the own writing device dependent software.

CADAM (Computer Graphics Augmented Design And Manufacturing) is the CAD system used and its features are mentioned. That system is linked to the FEM program SAP4. From the nine finite elements of this program, three are fully supported by the link so far: beam, thin shell/plate and boundary element.

The preprocessing uses the 3-D-MESH-module of CADAM to elaborate the data of the FEM-model and to store them in the database. Hereout a meshfile is obtained. On the other hand, macro-programs have been developed. At the end of each macro the gathered data are written to macro-files. These, together with the meshfile, provide the input for the SAP4-interface., a user written link that creates a formatted SAP4-inputfile.

SAP4 produces outputfiles for nodal displacements and element stresses only. The conversion of numerical results in principal stresses and other items and the graphical displays require postprocessing.

For the preferred postprocessing possibility, macro's have been written such that the Interface to CADAM Database of the Geometry Capability can yield even complete drawings, i.e. vectorplots and contourplots.

Two applications are presented: an assembly of four hypar shells as a model for simple roof structures for medical health centers in developing countries; a 7m by 7m plate with a liftshaft opening and supporting two point loads. The plots shown prove the capability of the developed linking of a general CAD system with a FEM program.

Further expansion towards integration of manufacturing modules lies still open, what can be problematical with so called closed FEM packages.

Three references cover reviews of pre- and postprocessor programs and a work on the use of a CAD system for pre- and postprocessing of FEM analysis.

5.5 **D. Van Gemert, C. Vanoverschelde** and **M. Vanden Bosch** present a mathematical model for calculation of rebuild of a R/C mushroom slab in view of the strenghtening of the plate by means of externally bonded (glued) steel strips.

In the plates of a factory, openings had to be provided and existing openings closed. A finite element mesh was elaborated in such a way that comparison of the computation results with those of a finite difference analysis were easy. Also a frame analysis was performed.

The general picture of finite difference and finite element calculations looks the same. However, individual moment values could differ considerably in certain places: the FEM yielded the conservative values while in other places it was the finite difference method. That happened as well for positive as for negative moments.

Altogether, the results were favourable for a strenghtening procedure by means of epoxy bounded external steel strips, because all the necessary strenghtening works could be done at the bottom side of the plate.

In conclusion it is emphasized that a considerable amount of experience is needed to find the most appropriate finite element distribution and that in many cases it is better to prefer a more refined mesh above more interpretation costs.

The references cover the following topics: design of externally bonded steel reinforcements in bending; practical computation of orthotropic plates.

5.6 **Paul Lew, Abraham Gutman, Leonid Zborovsky** and **Lee Petrella** present the design and the analysis of the Winter Garden, which is the centerpiece of the World Financial Center in New York City and thus one of the key indoor spaces in this city.

A telescoping barrel vault roof, a half dome, a large arch folded plate front wall (one of the largest glass walls ever built) and a U-shaped stiffening ring are structural elements of a complex assemblage.

Instead of doing one large complex analysis, the structural engineers elected to subdivide the structure into basic elements and analyze the interaction of these elements by controlled forces and displacements.

A description of the structural elements is given. The four telescoping barrel vaults or arch vaults are space trusses, each of fifteen feet long. They have the half circular form raised some thirteen feet on vertical straight parts at their bottoms. One vault spans 110 feet, two 90 and the last 70, which is situated at the end of the row. There the volume is closed by a half dome. In the front is a 100 feet wide by 115 feet high semi-circular glass wall. To span this expanse, the front wall was shaped into a folded plate with in-plane trusses.

The U-shaped stiffening ring takes the thrusts of the vaults and fulfills other functions.

Bridging and intertruss struts limit the relative movement of the telescoping vaults lateral out of plane movement.

These vaults are examined at first in the main direction of span and to this effect modeled with linear axial straight line truss chords. Two issues were considered separately from the basic truss analysis: a) the effect of the displacement of the stiffening ring at the arch supports; b) the effect of the top and bottom chords of the truss being continuous and curved rather than straight and hinged at the joints. The first concerned 2 inches displacement of the ring and proved to be neglectable. The second was studied in one computer run, that was modelled as a frame with curved sections between truss panel points and yielded a ratio between the arches axial forces and bending moment in the top and bottom chords. This ratio was then applied to all the other arch or vault trusses.

Then these arch vaults were examined perpendicular to the main span. They are consequently braced following the cylindrical vault. Such curved bracing has an overturning effect.

To limit the relative movement of each telescoping vault relative to each other, it was realized that the end dome was rigid and it was treated as a support of a frame system of purlins, spanning between the main trusses and down the steps in the telescoping vaults.

The edge of the dome was supported by a double vault truss at the step between the last vault and the end dome.

The front wall folded plate has inclined faces with build-in trusses which span to a Vierendeel horizontal truss about 78 feet above the base. The front wall was laterally supported by the diaphragm truss of the arch vault roof. To determine the effect of displacements of the arch vault on the front wall, the supports of the front wall that would belong to the arch roof were displaced the amount they would be displaced from the roof. The forces and moments that resulted in the front wall from these displacements, were added to the primary forces and moments from loads.

The stiffness of the U-shaped stiffening ring was the key element allowing other structural elements to be light by providing them adequate boundary conditions. The stiffness was enhanced to its threefold by means of its connection to a concrete roof deck, as computed by a special run, done by adding an in-plane infill finite element.

In the conclusion is underlined the benefit of subdividing the structure in structural systems in order to compute them separately and to consider their interaction. The understanding of the primary forces on the structure and the interaction of the various structural systems would have been lost, if the structure had been analyzed in one massive run.

5.7 **G. De Roeck** and **C. Vanoverschelde** computed and tested a foamed hypar.

The frame consists primarily of lattice girders made of galvanized steel assembled such as to form a horizontal grid structure with square meshes. As such an assemblage has nearly no torsional stiffness, it is very easily twisted into the form of a hypar. The thus obtained spatial system is provided with cardboard filled galvanized wire mesh. This cardboard becomes a support for mortar/plaster and principally a thick polyurethane foam sprayed in situ.

The mean density of the foam is 62kg/m3. The elastic charateristics measured in the laboratory correspond very well with other tests made by the producers of the foam. For the numerical model, the value E=14N/mm2 and a zero Poisson coefficient were retained.

The lattice girders could, according to experimental investigation, be modelled approximately as massive beams if they are long enough.

The tests (on two models) have revealed that the spacing of lattice girders is of little consequence. The point loads in the free nodes simulated a uniformly distributed load of about 1700N/m2. After unloading, the shell nearly gets back its original shape.

A linear and a nonlinear computation with the TITUS program were performed: the nonlinear behavior was found as most pronounced in the region of great displacements; the principal stress directions may vary considerably, but without significant change in the value of the principal stresses.

There is a reasonable agreement between the results of the numerical analysis and the experimental results in the lower range of the load.

The observed ultimate load corresponds to moderate stresses. Thus excessive deformation (buckling, snap-through behavior) is the failure cause.

The combination of polyurethane foam and lattice girders is used successfully for walls and can be extended to build roof structures, if the following requirements are fulfilled :

-the geometry of the polyurethane structural elements has to contribute to the stiffness;

-the supports must be very stiff and tensioner rods provided;

-the boundaries have to be provided with extra lattice girders.

It is noted that the pitch between the girders has little influence. In the case of moderate displacements, a linear model can fairly predict structural behavior.

The references cover simple housing, thin shell structures and more specifically the hypar, the user's manual of the computer program and studies concerning the assembling of the considereds building materials, i.e. the polyurethane and the lattice girders.

5.8 G. Valente and M. Moscetti present a numerical analysis of tensile structures with minimum surface based on survey of soap film models.

Soap film models are obtained on a special frame which reproduces the contour of the structure, in scale. The soap film surface is a minimal area surface.

The soap film is instable in time. Its transparency makes it almost impossible to photograph.

The maximum error as a total of model and photographic errors is about 10cm in the surface height. Starting from the values obtained by the photographic survey, a numerical method of mathematical programming yields very accurate values for this elevation.

At first and mainly,is considered the surface as a function of the z-coordinates (elevation) corresponding to fixed points in the ground plan (x- and y-values).

The function minimization (in the neighbourhood of the required minimum height) by conjugate gradients and the methods using conjugate directions are exposed.

The method of successive linear searches is quadratically convergent when using any set of A-conjugate directions, where A, the matrix of second-order partial derivatives of the area, is symmetric and positive definite.

The method of conjugate gradients yields a process which, apart from rounding errors, locates the minimum of any quadratic function of n arguments in at most n steps.For functions which are not quadratic, the process is iterative rather than n-step, and a test of convergence is required.

For quadratic functions, any choice of starting values is in principle equally satisfactory. For general functions, the best to avoid problems is to start from the numerical values obtained from the photographic survey of the soap film.

An application to a hypar yields elevations slightly superior to those measured from the soap film, but coincide practically with the values computed from a finer mesh.

Then a couple of applications to surfaces that cannot be defined by an equation are presented. In the model, the boundary cables are represented by silk threads which are drawn in by the surface tension of the soap film. Again, the computed values are higher as the experimental.

In any case, the survey of the soap film in conjunction with the presented numerical analysis yields a powerful tool for the new problem raised by tensile structures.

Only the variability of elevation is considered presently and this problem is solved by locating an unconstrained minimum of a function of several (independant) variables z. It is possible to solve this problem with other boundary conditions, such as a boundary cable of preassigned length, assuming as other variables the abscissae x and y and expressing several of them as dependant on others.

References cover an article on function minimization by conjugate gradients, an abstract of the doctoral thesis of the second author on the optimization of tensile structures (in Italian language) and the paper presented by the authors at the IASS Congress on large span structures in Moscow (1985).

Acknowledgments

Professor H. Rühle advocated the organizing of the meeting of the IASS-Group Numerical Methods in the University of Louvain (K.U.Leuven) and Professor R.Palanco agreed very friendly.

The Board of Executives of the IASS showed great interest in the papers presented and as a consequence the organizers of the IASS-Symposium in Osaka (1986) asked us to write this paper.

Professor A. Deruyttere, Vice-Rector of the University of Louvain, send us a copy of the AGARD Conference Proceedings No. 397 on "Mechanical Qualification of Large Flexible Spacecraft Structures" (cited hereabove under no. 2.1).